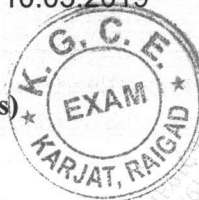


(3 Hours)



[Total marks : 80

- Note :-**
- 1) Question number 1 is **compulsory**.
 - 2) Attempt any **three** questions from the remaining **five** questions.
 - 3) **Figures** to the **right** indicate **full** marks.

- Q.1 a) If $u = \log\left(\frac{x}{y}\right) + \log\left(\frac{y}{x}\right)$, find $\frac{\partial u / \partial x}{\partial u / \partial y}$ 03
- b) Find the value of $\tanh(\log x)$ if $x = \sqrt{3}$ 03
- c) Evaluate $\lim_{x \rightarrow 3} \left[\frac{1}{x-3} - \frac{1}{\log(x-2)} \right]$ 03
- d) If $u = r^2 \cos 2\theta$, $v = r^2 \sin 2\theta$, find $\frac{\partial(u,v)}{\partial(r,\theta)}$ 03
- e) Express the matrix $A = \begin{pmatrix} 2+3i & 2 & 3i \\ -2i & 0 & 1+2i \\ 4 & 2+5i & -i \end{pmatrix}$ as the sum of a Hermitian and a Skew-Hermitian matrix. 04
- f) Expand $\tan^{-1}x$ in powers of $\left(x - \frac{\pi}{4}\right)$ 04
- Q.2 a) Expand $\sin^7\theta$ in a series of sines of multiples of θ 06
- b) If $y = \sin^2x \cos^3x$, find y_n 06
- c) Find the stationary values of $x^3 + y^3 - 3axy$, $a > 0$ 08
- Q.3 a) Compute the real root of $x \log_{10}x - 1.2 = 0$ correct to three places of decimals using Newton-Raphson method. 06
- b) Show that the system of equations $2x - 2y + z = \lambda x$, $2x - 3y + 2z = \lambda y$, $-x + 2y = \lambda z$ can possess a non-trivial solution only if $\lambda = 1, \lambda = -3$. Obtain the general solution in each case. 06
- c) If $\tan(\alpha + i\beta) = \cos\theta + i \sin\theta$, prove that $\alpha = \frac{n\pi}{2} + \frac{\pi}{4}$ and $\beta = \frac{1}{2} \log \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$ 08

- Q. 4 a) Using the encoding matrix as $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, encode and decode the message MOVE 06
- b) If $u = f(e^{x-y}, e^{y-z}, e^{z-x})$ then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ 06
- c) If $y = a \cos(\log x) + b \sin(\log x)$, then show that $x^2 y_{n+2} + (2n + 1)xy_{n+1} + (n^2 + 1)y_n = 0$ 08
- Q. 5 a) If $1, \alpha, \alpha^2, \alpha^3, \alpha^4$, are the roots of $x^5 - 1 = 0$, find them and show that $(1 - \alpha)(1 - \alpha^2)(1 - \alpha^3)(1 - \alpha^4) = 5$ 06
- b) If $\theta = t^n e^{-r^2/(4t)}$, Find n which will make $\frac{\partial \theta}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \theta}{\partial r} \right)$ 06
- c) Find the root (correct to three places of decimals) of $x^3 - 4x - 9 = 0$ lying between 2 and 3 by using Regula-Falsi method. 08
- Q. 6 a) Find non-singular matrices P and Q such that $A = \begin{pmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{pmatrix}$ is reduced to normal form. Also find its rank. 06
- b) Find the principle value of $(1 + i)^{1-i}$ 06
- c) Solve the following equations by Gauss-Seidel method 08
- $$27x + 6y - z = 85$$
- $$6x + 15y + 2z = 72$$
- $$x + y + 54z = 110$$
- (Take three iterations)